

CP Violation in Neutrino Oscillations in Matter

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Abstract

We estimate CP violation for several experimental facilities studying neutrino oscillations. We also estimate the probability of ν_μ to ν_e conversion, using values of the parameter θ_{13} within the known limits, in order to suggest new experiments to measure CP violation for neutrinos moving in matter.

1 Introduction

The study of CP violation (CPV) is essential for understanding weak interactions. Almost half a century ago CP violation in weak interactions was found in the decay of K_L^0 into $\pi^+ + \pi^-$ [1] and $2\pi^0$ [2], with branching ratios of the order of .001. The decay $K_L^0 \rightarrow \pi^0 + \nu + \bar{\nu}$ is almost entirely CP violating [3] but requires accurate determination of the CKM matrix [4] and accurate measurements. See Ref[5] for a review of this experiment and references. There have many other studies of CP asymmetries in weak decays: see Ref[6] for a recent study of \bar{B} radiative decay with references to earlier work on CP violation in various weak decays.

In our present work we study possible CP and CPV measurements using neutrino oscillations. In recent years there have been a number of experimental studies of neutrino oscillations using neutrino beams from accelerators and reactors, and a most important objective of these experiments is the measurement of CP violation (CPV). In our present work on estimating CPV we use parameters for the baseline and energy corresponding to MiniBooNE[7], JHF-Kamioka [8], MINOS[9], and CHOOZ[10], which are on-going projects.

There have been many recent studies of CP and T symmetries via neutrino oscillations for future facilities, e.g., see Refs[11, 12], which also give references to earlier publications, and the ISS report[13] on future neutrino facilities. One possible future facility for studying CPV and the δ_{CP} parameter is the LBNE Project, where neutrino beams produced at Fermilab would have a baseline of $L \simeq 1200$ km, being detected with deep underground detectors[14, 15]. With the methods used in the present work, described below, predictions of CPV with the baseline and energies of the LBNE Project have recently been made for δ_{CP} from 90 to 0 degrees[16]. The angle θ_{13} is not well known, and will be measured by the Daya Bay experiment in China. First, we use $\delta_{CP} = 90^\circ$ and two values of θ_{13} to explore the dependence of CP and CPV neutrino oscillations on this parameter for all on-going projects. We then calculate CPV for JFK-Kamioka baseline with $E=.48$ GeV, which has a large CPV for $\sin\theta_{13}=0.19$ (Sec. 3), to estimate CPV for values of θ_{13} expected to be found (Sec. 4).

A major complication for the determination of T, CP, and CPT violation is the interaction of neutrinos with matter as they travel along the baseline. These matter effects have been studied by a number of theorists. See, e.g., Refs[17, 18, 19]. The main objective of the present research is to estimate matter effects for CPV. for the MiniBooNE, JHF-Kamioka, MINOS, and CHOOZ facilities.

For the basic interactions, which are CPT invariant for local theories, CP violation also implies T violation. Our present research is an extension of our recent work on T reversal violation[20]. In that study we used the formalism of Ref[21] for $\nu_e \leftrightarrow \nu_\mu$ TRV, and that of Ref[22] for $\nu_e \rightarrow \nu_\mu$ conversion probability to calculate the effects of neutrinos moving through matter. In the present work we use the notation and formalism of Jacobson and Ohlsson[23], who studied possible matter effects for CPT violation.

CP violation in the $a - b$ sector is given by the transition probability, denoted by $\mathcal{P}(\nu_a \rightarrow \nu_b)$, for a neutrino of flavor a to convert to a neutrino of flavor b ; and similarly for antineutrinos $\bar{\nu}_a, \bar{\nu}_b$. The CPV probability differences (note that the C operator changes a particle to its antiparticle) are defined as

$$\Delta\mathcal{P}_{ab}^{CP} = \mathcal{P}(\nu_a \rightarrow \nu_b) - \mathcal{P}(\bar{\nu}_a \rightarrow \bar{\nu}_b) . \quad (1)$$

In our present work we study $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ and $\mathcal{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, since the neutrino beams at MiniBooNE, JHF-Kamioka, and MINOS, as well as most other experimental facilities, are muon or anti-muon neutrinos.

2 Transition Probability $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$

In this section we review the derivation of the probability of a muon neutrino to convert to an electron neutrino, $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$, using the notation of Ref[23]. We then make an estimate of the transition probabilities for sample accelerator and reactor experiments. Although at the present time no experiments for CPV are possible, this can serve as a basis for future experiments. In the next section we give somewhat more accurate calculations for CPV for the same set of experimental facilities.

As in Refs[21, 23] we use the time evolution matrix, $S(t, t_0)$ to derive the transition probabilities. For neutrino oscillations the initial neutrino beam is emitted at time t_0 , usually taken as $t_0 = 0$, and the neutrino or converted neutrino is detected at baseline length $= L$ at time $= t$. Since the neutrinos move with a velocity near that of the speed of light, at the end of our derivation we take $t - t_0 \rightarrow L$, with the units $c=1$.

Given the Hamiltonian, $H(t)$, for neutrinos, the neutrino state at time $= t$ is obtained from the state at time $= t_0$ from the matrix, $S(t, t_0)$, by

$$|\nu(t)\rangle = S(t, t_0)|\nu(t_0)\rangle \quad (2)$$

$$i\frac{d}{dt}S(t, t_0) = H(t)S(t, t_0) . \quad (3)$$

Neutrinos (and antineutrinos) are produced as ν_a , where a is the flavor, $a = e, \mu, \tau$. However, neutrinos of definite masses are ν_α , with $\alpha = 1, 2, 3$. The two forms are connected by a 3 by 3 unitary transformation matrix, U : $\nu_a = U\nu_\alpha$, where ν_a, ν_α are 3x1 column vectors and U is given by ($\sin\theta_{ij} \equiv s_{ij}$)

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} ,$$

similar to the CKM matrix for quarks. We use the best fit value[11] $s_{23} = 0.707$. θ_{13} is not well known. We use $s_{12} = 0.56$. We use $s_{13} = 0.19$, consistent with a recent analysis[12], and $s_{13}=0.095$, to determine the dependence of CP and CPV on this parameter, which is not well known. The CP phase δ_{CP} is also not well known. For simplicity we choose $\delta_{CP} = \pi/2$, and calculate the dependence of $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ on δ_{CP} , as discussed below.

In the vacuum the $S(t, t_0)$ is obtained from

$$S_{ab}(t, t_0) = \sum_{j=1}^3 U_{aj} \exp^{iE_j(t-t_0)} U_{bj}^* . \quad (4)$$

Since neutrino beams in neutrino oscillation experiments travel through matter, and the main neutrino-matter is scattering from electrons, we must include potential, $V = \sqrt{2}G_F n_e$, for neutrino electron scattering in the earth: where G_F is the universal weak interaction Fermi constant, and n_e is the density of electrons in matter. Using the matter density $\rho=3$ gm/cc, the neutrino-matter potential is $V = 1.13 \times 10^{-13}$ eV.

The transition probability $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ is obtained from S_{12} , with $\mathcal{P}(\nu_\mu \rightarrow \nu_e) = |S_{12}|^2 = \text{Re}[S_{12}]^2 + \text{Im}[S_{12}]^2$, with

$$S_{12} = c_{23}\beta - is_{23}ae^{-i\delta_{CP}}A \quad (5)$$

$$a = s_{13}(\Delta - s_{12}^2\delta) \quad (6)$$

$$\delta = \delta m_{12}^2/(2E) \quad (7)$$

$$\Delta = \delta m_{13}^2/(2E) \quad (8)$$

$$A \simeq f(t)I_\alpha * \quad (9)$$

$$I_\alpha * = \int_0^t dt' \alpha^*(t')f(t') \quad (10)$$

$$\alpha(t) = \cos\omega t - i\cos 2\theta \sin\omega t \quad (11)$$

$$f(t) = e^{-i\bar{\Delta}t} \quad (12)$$

$$2\omega = \sqrt{\delta^2 + V^2 - 2\delta V \cos(2\theta_{12})} \quad (13)$$

$$\beta = -i\sin 2\theta \sin\omega L \quad (14)$$

$$\bar{\Delta} = \Delta - (V + \delta)/2 \quad (15)$$

$$\sin 2\theta = s_{12}c_{12}\frac{\delta}{\omega}, \quad (16)$$

where the neutrino mass differences are $\delta m_{12}^2 = 7.6 \times 10^{-5}(eV)^2$ and $\delta m_{13}^2 = 2.4 \times 10^{-3}(eV)^2$. Note that $\delta \ll \Delta$, and $t \rightarrow L$ for $v_\nu \simeq c$. From Eqs.(17,5):

$$\begin{aligned} \text{Re}[S_{12}] &= s_{23}a[\cos((\bar{\Delta} + \delta_{CP})L)\text{Im}[I_{\alpha*}] - \sin((\bar{\Delta} + \delta_{CP})L)\text{Re}[I_{\alpha*}]] \\ \text{Im}[S_{12}] &= -c_{23}\sin 2\theta \sin\omega L - s_{23}a[\cos((\bar{\Delta} + \delta_{CP})L)\text{Re}[I_{\alpha*}] \\ &\quad + \sin((\bar{\Delta} + \delta_{CP})L)\text{Im}[I_{\alpha*}]] \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{P}(\nu_\mu \rightarrow \nu_e) &\simeq (c_{23}s_{12}c_{12}(\delta/\omega)\sin\omega L)^2 + (s_{23}s_{13}\sin\bar{\Delta}L)^2 \\ &\quad + (s_{23}s_{13}(\cos\bar{\Delta}L - 1))^2 + 2s_{13}s_{12}c_{12}s_{23}c_{23} \\ &\quad (\delta/\omega)\sin\omega L(\cos\bar{\Delta}L - 1). \end{aligned} \quad (18)$$

From Eq(18) we obtain the results for $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ shown in Fig.1.

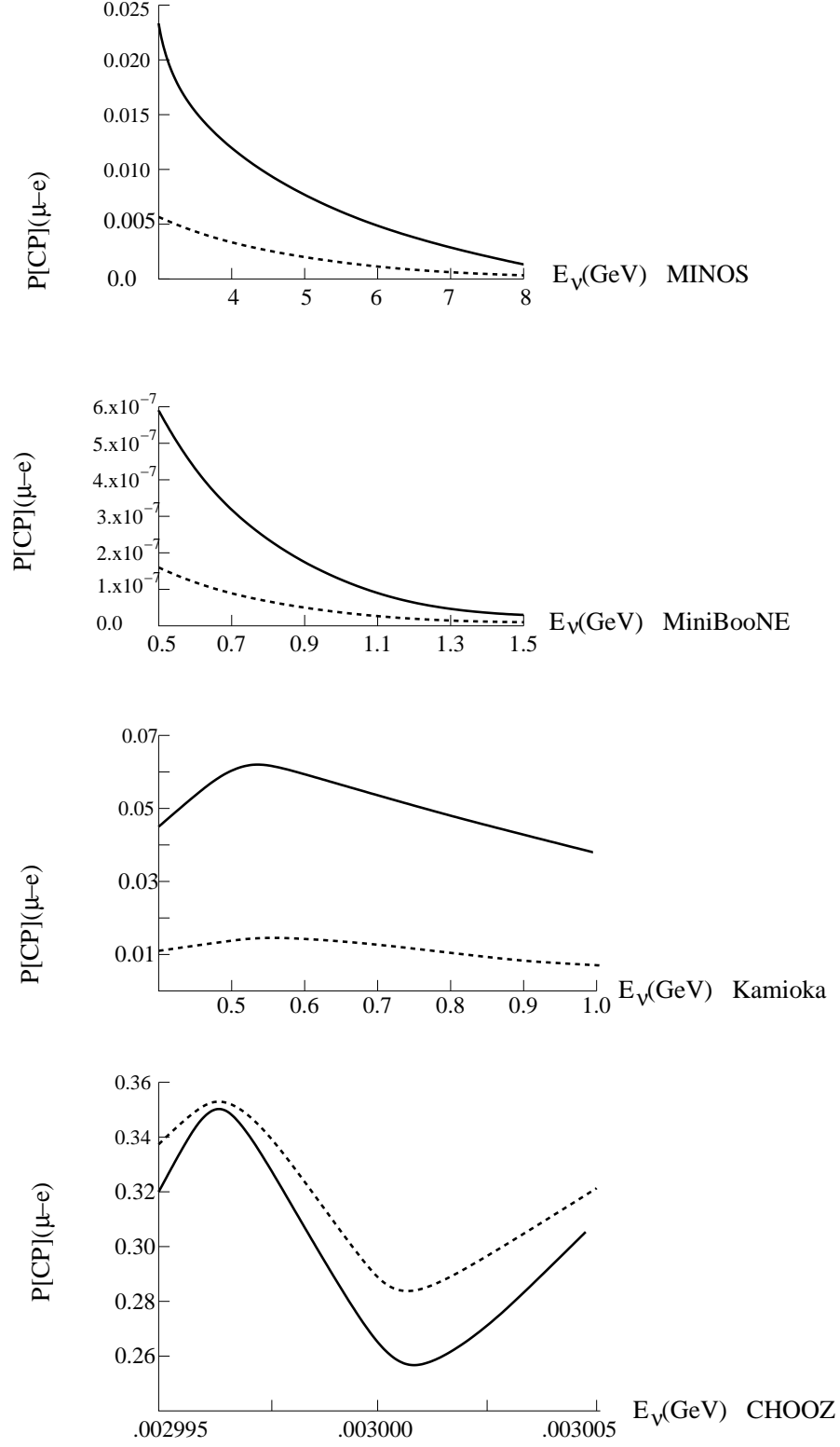


Figure 1: The ordinate is $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ for MINOS($L=735$ km), MiniBooNE($L=500$ m), JHF-Kamioka($L=295$ km), and CHOOZ($L=1$ km). Energy= E in GeV. Solid curve for $s_{13}=0.19$ and dashed curve for $s_{13}=0.095$

Although these results are only for muon to electron neutrino conversion, they can provide guidance for future experiments on CPV via $\nu_\mu \leftrightarrow \nu_e$ oscillation. Note that in Ref[8] $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ was calculated for the 295 km JHF-Kamioka project for $E=0-2$ GeV, and our calculation based on the theory developed in Refs.[23, 21], finds that with similar parameters $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ is in agreement for $E=.4-1.0$ GeV with this earlier estimate. Since there is uncertainty in the value of δ_{CP} , we calculated $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ for $\delta_{CP} = 0$. We do not show the results, as they are almost the same as shown in Fig. 1.

3 CP Violation $\Delta\mathcal{P}_{\mu e}^{CP}$

In this section we shall extend the derivation of the transition probability $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ of the previous section to derive the CPV probability

$$\begin{aligned}\Delta\mathcal{P}_{\mu e}^{CP} &= \mathcal{P}(\nu_\mu \rightarrow \nu_e) - \mathcal{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\ &= |S_{12}|^2 - |\bar{S}_{12}|^2\end{aligned}\quad (19)$$

with S_{12} defined in Eq(8) and

$$\bar{S}_{12} = c_{23}\bar{\beta} - is_{23}ae^{i\delta_{CP}}\bar{A}, \quad (20)$$

with $\bar{\beta} = \beta(V \rightarrow -V)$ and $\bar{A} = A(V \rightarrow -V)$. For example (see Eqs(16,18)) $2\bar{\omega} = \sqrt{\delta^2 + V^2 + 2\delta V \cos(2\theta_{12})}$ and $\bar{\Delta} = \Delta + (V - \delta)/2$. Using conservation of probability[23], $|A|^2 = |\bar{A}|^2$. With $\delta_{CP} = \pi/2$, $e^{(-/+)\delta_{CP}} = (-/+)i$.

$$\Delta\mathcal{P}_{\mu e}^{CP} = c_{23}^2(|\beta|^2 - |\bar{\beta}|^2) - 2c_{23}s_{23}a(Im[-i\beta A^*] - Im[i\bar{\beta}\bar{A}^*]). \quad (21)$$

From Eq(21), the definitions in the previous section, defining $s \equiv \sin(\omega L)$, $c \equiv \cos(\omega L)$, and using $\delta_{CP} = \pi/2$ one finds

$$\begin{aligned}\Delta\mathcal{P}_{\mu e}^{CP} &= c_{23}^2 s_{12}^2 c_{12}^2 \delta^2 \left(\frac{s^2}{\omega^2} - \frac{\bar{s}^2}{\bar{\omega}^2} \right) + 2c_{23}s_{23}s_{12}c_{12}s_{13}\delta(\Delta - \delta s_{12}^2) \\ &\quad \left(\frac{s}{\omega}(c - \cos\bar{\Delta}L)\frac{\bar{\Delta} - \omega\cos 2\theta}{\bar{\Delta}^2 - \omega^2} + \frac{\bar{s}}{\bar{\omega}}(\bar{c} - \cos\bar{\Delta}L)\frac{\bar{\Delta} - \bar{\omega}\cos 2\bar{\theta}}{\bar{\Delta}^2 - \bar{\omega}^2} \right).\end{aligned}\quad (22)$$

The results for $\Delta\mathcal{P}_{\mu e}^{CP}$ are shown in Fig.2. Note that the largest values for CPV are for CHOOZ, with a small baseline and low energy. With Kamioka parameters, CPV would also be a few percent, which might be measurable. However, for experimental tests of CPV one needs both ν_μ and $\bar{\nu}_\mu$ beams with the same parameters. Perhaps this will be possible in the future.

As has been stated in many publications, in vacuum $\Delta\mathcal{P}_{\mu e}^{CP}$ is given by $\Delta\mathcal{P}_{\mu e}^T$, and both vanish if $\delta_{CP} = 0$. However, with matter effects ($V \neq 0$), $\Delta\mathcal{P}_{\mu e}^{CP}$ and $\Delta\mathcal{P}_{\mu e}^T$, must be treated separately, as we have done. The magnitude of δ_{CP} is important for predictions, and it is expected that in the future it will be determined with greater accuracy.

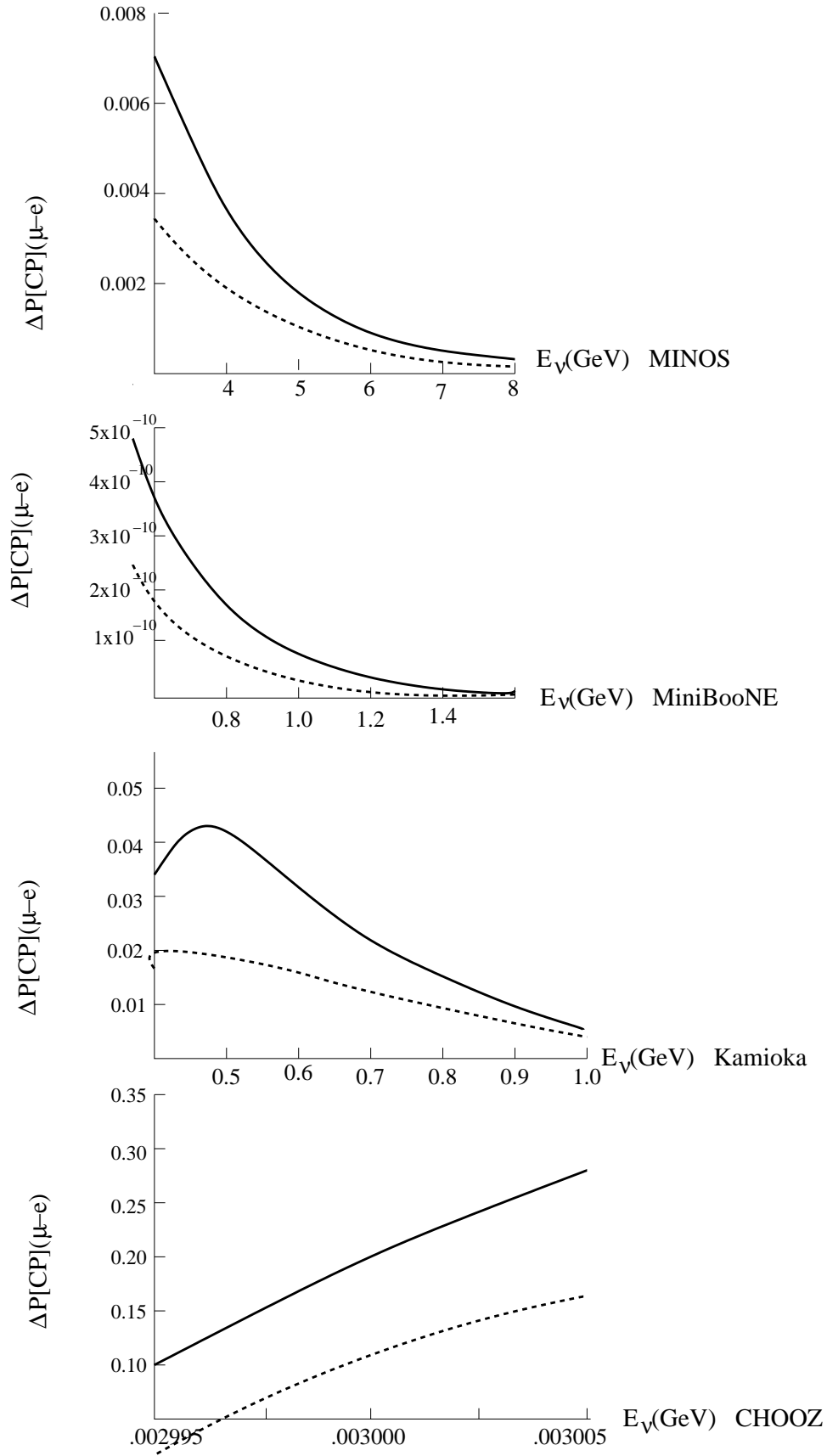


Figure 2: The ordinate is $\Delta\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ for MINOS(L=735 km), MiniBooNE(L=500m), JHF-Kamioka(L=295 km), and CHOOZ(L=1 km). Energy=E in GeV. Solid curve for $s_{13}=0.19$ and dashed curve for $s_{13}=0.095$

4 $\Delta\mathcal{P}_{\mu e}^{CP}$ For JHF-Kamioka with E=0.48 GeV and $\sin\theta_{13}=0.0$ to 0.19

From Fig. 2 one sees that with the value $\sin\theta_{13} = 0.19$ CPV reaches a value of over 4%, for the JHF-Kamioka project, which could be detected if beams of both neutrinos and antineutrinos were available. Since the value of $\sin\theta_{13}$ is not known at the present time, we use the energy and baseline values of E=0.48 GeV, L=295 km with $\sin\theta_{13}$ from 0.0 to 0.19[12]. Using Eq.(22), with $\sin\theta_{13}$ a variable, one obtains the results are shown in Fig. 3.

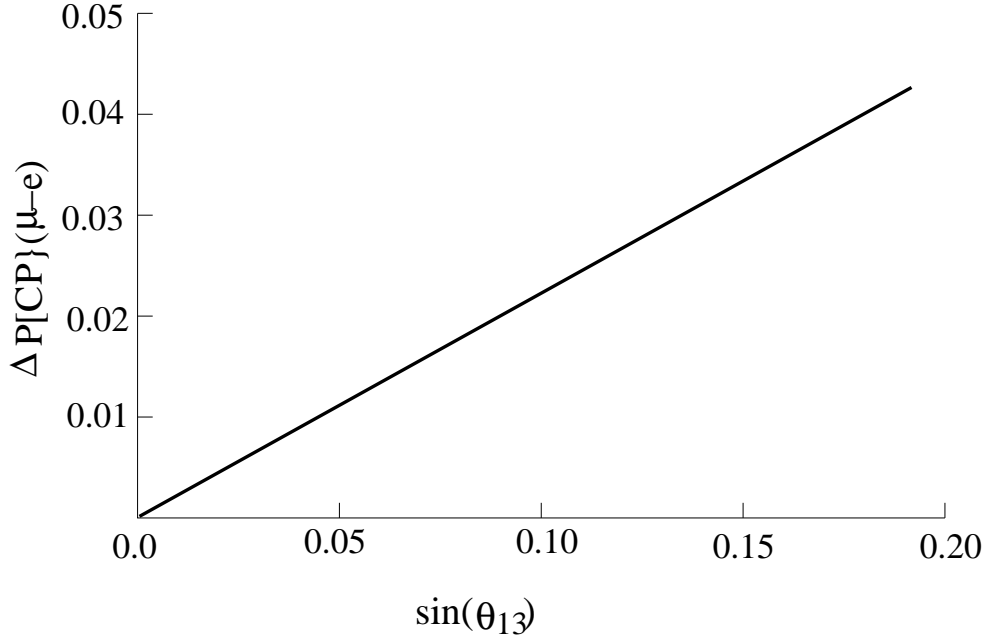


Figure 3: $\Delta\mathcal{P}(\nu_{\mu} \rightarrow \nu_e)$ for JHF-Kamioka(L=295 km), E=0.48 GeV, as a function of $\sin\theta_{13}$

Note that since the second term in Eq.(22) is dominant, the dependence of $\Delta\mathcal{P}(\nu_{\mu} \rightarrow \nu_e)$ on $\sin\theta_{13}$ is almost linear.

5 Conclusions

We have estimated CP violation for a variety of experimental neutrino beam facilities. No experiments are possible now to test CPV via neutrino oscillations, since beams of both neutrino and antineutrino with the same flavor would be needed, with parameters chosen for a CPV of 1% or more to make the experimental measurement possible. Our results should help in planning future experiments. Note, however, that our results depend on the value of s_{13} and δ_{CP} , which are not well known, and we have used two values for s_{13} to determine the dependence of CP and CPV on this parameter (see Fig. 2), and estimated CPV as a function of s_{13} for the JFK-Kanioka baseline of 295 km and energy $E=0.48$ GeV, with the CPV probability over 4% for $s_{13}=0.19$ (see Fig. 3).

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